

Snow College Jr. Mathematics Contest

April 5, 2022

Junior Division: Grades 7–9

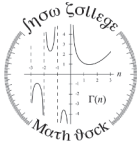
Form: T

Please read all instructions on this page very carefully.

1. Leave this booklet closed until you are instructed to begin.
2. Bubble your test version (Form T) on the upper left side of the bubble sheet. Make sure your bubble sheet has your name already printed (if not, return to the registration table).
3. This is a one hour examination consisting of 20 multiple choice problems. Bubble in the single best answer to each question you choose to answer clearly on the answer sheet. Completely erase any answers you wish to change.
4. Avoid random guessing as there is a penalty for wrong answers. There is no penalty for leaving a question blank. The formula for scoring the test is $\text{Score} = 20 + 4R - W$ where R and W denote the number right and wrong respectively. The possible scores range from 0 to 100.
5. Ties will be broken by the first discrepancy in the following five problems *in order*: 1, 5, 7, 14, 20. In the event of no discrepancies in those problems, the tie will be broken by a coin toss.
6. No calculators are allowed. Diagrams are not necessarily drawn to scale.
7. Do not talk or disrupt other test takers during the exam. Cell phones must be OFF (not just on silent or vibrate, but OFF). No ear buds are allowed.
8. Please raise your hand if you need scratch paper or a new pencil; a proctor will assist you.
9. The proctors have been advised to not answer questions pertaining to the exam.
10. If you have time we recommend you recheck your answers. If you finish early you may quietly turn your answer sheet in and wait for the group competition. Please don't leave. After the one hour time limit is up the proctors will call for all answer sheets; hand them in promptly.

After the test:

1. You may keep this test booklet and the pencils. There is a group activity during the 2nd hour.
2. If you RSVP'd to spend time with one of our science departments for lunch, please meet them in the science center (GRSC) or library auditorium; otherwise lunch may be purchased at the Snow College Cafeteria or downtown. In any event, you should plan to be back at the LDS Institute by 1:00 p.m. for the scores and presentation of the awards.
3. The top scorers from each grade 7–9 will be awarded a prize.
4. Thanks for coming; we hope you had fun and learned some math. Your instructors will be happy to work the problems for you. They will also be given copies of your answer sheets.



Snow College Jr. Mathematics Contest

key

April 5, 2022

Junior Division: Grades 7–9

Form: T

Bubble in the single best choice for each question you choose to answer.

1. The sum of three consecutive integers is equal to the product of those integers. How many sets of integers satisfy this condition?

(A) 0
(B) 1
(C) 2
 (D) 3 {−3, −2, −1}, {−1, 0, 1}, {1, 2, 3}
(E) 5

SOLN Let x be the middle number. The three are then $x - 1$, x , and $x + 1$. So

$$(x - 1)(x)(x + 1) = x - 1 + x + x + 1$$

$$x^3 - x = 3x$$

$$x^3 - 4x = 0$$

$$x(x^2 - 4) = 0$$

$$x(x - 2)(x + 2) = 0$$

$$x = 0, 2, -2 \quad \square$$

2. The regular price of a pair of jeans is \$60. They are on sale for $1/4$ off the regular price. If you buy the jeans on sale and pay 8% sales tax, what is the total cost?

(A) \$16.20
(B) \$45.80
 (C) \$48.60
(D) \$51.84
(E) \$62.62

SOLN $(\$60 \cdot \frac{3}{4}) \cdot 1.08 = \$60 \cdot 3 \cdot \frac{1.08}{4} =$
 $\$60 \cdot 3 \cdot 0.27 = \$60 \cdot 0.81 = \$48.60 \quad \square$

3. What is the surface area of a rectangular solid with a length of 7 cm, a width of 3 cm, and a height of 5 cm?

(A) 108 cm²
(B) 126 cm²
(C) 138 cm²
 (D) 142 cm²
(E) 166 cm²

SOLN $2(5 \times 7) + 2(5 \times 3) + 2(3 \times 7) = 142 \quad \square$

4. Suppose the Earth is a perfect sphere and that there is a steel belt fitting snugly around it at the equator. If the length of the belt were increased by 10 feet, how far above the Earth would the belt be raised if it remained circular and centered around the Earth?

(A) Less than 1 inch
(B) Between 1 inch and 2 inches
(C) Between 2 inches and 1 foot
 (D) Between 1 foot and 2 feet
(E) More than 2 feet

SOLN $\Delta C = 2\pi\Delta r$ so $\Delta r = \Delta C/2\pi =$
 $10 \text{ ft}/2\pi \approx 0.6366 \text{ ft.} \quad \square$

5. What is $1 + 3 + 5 + \dots + 497 + 499$?

(A) 62,000
 (B) 62,500
(C) 124,500
(D) 125,000
(E) 249,500

SOLN $1 + 499 = 500, 3 + 497 = 500, \dots$
 $249 + 251 = 500$. There are $249/2 + 0.5 =$
 125 pairs that sum to 500. \square

6. Clara has a tricycle with a large front wheel with a radius of 9 inches. The small rear wheels have a radius of 6 inches. The tricycle moves in a straight line with the large wheel making 150 revolutions. How many revolutions does each small wheel make?

- (A) $66\frac{2}{3}$
 (B) 150
 (C) 200
 (D) 225
 (E) 250

SOLN The smaller wheel makes $9/6 = 1.5$ times as many revolutions. \square

7. A problem on Sam's math test is stated as

$$\begin{array}{r} \text{O N E} \\ + \text{O N E} \\ \hline \text{T W O} \end{array}$$

If none of the letters represents either a 1 or 2 and each letter is a distinct digit, what are the values of T, W, and O?

- (A) $T = 7, W = 4, O = 3$
 (B) $T = 8, W = 3, O = 4$
 (C) $T = 8, W = 4, O = 3$
 (D) $T = 9, W = 4, O = 6$
 (E) $T = 9, W = 3, O = 4$

SOLN $E + E$ is even, so O must be even. Since $O + O$ is a single digit, $O = 4$. This means E is 2 or 7, but 2 is not allowed so $E = 7$. The value of N can be 3, 5, 6, 9, or 0. If N is 3, W is 7 which is already used. If N is 5, W is 1 which is not allowed. If N is 9, W is also 9. If N is 0, W is 1 which is not allowed. So $N = 6$ leading to $W = 3$ and $T = 9$. \square

8. $1.5 \text{ m} + 60 \text{ cm} + 0.02 \text{ km} =$

- (A) 0.221 m
 (B) 2.21 m
 (C) 22.1 m
 (D) 221 m
 (E) 0.0221 m

SOLN Since all answer choices are in meters, convert the last two addends to meters. $1.5 \text{ m} + 0.6 \text{ m} + 20 \text{ m} = 22.1 \text{ m}$ \square

9. Joe's income has been increasing each year by the same dollar amount. The first year his income was \$24,000, and the ninth year his income was \$31,680. In which year will his income be \$45,120?

- (A) 16th year
 (B) 19th year
 (C) 21st year
 (D) 22nd year
 (E) 23rd year

SOLN We have an arithmetic sequence with $a_1 = 24\,000$, $a_9 = 31\,680$. The formula for the n th term $a_n = a_1 + d(n - 1) \Rightarrow d = 960$. When $a_n = 45\,120$, $n = 23$. \square

10. If $\frac{1}{4x}$ equals the reciprocal of x^3 , then x could equal

- (A) $\frac{1}{8}$
 (B) $\frac{1}{2}$
 (C) 2
 (D) 8
 (E) 10

SOLN $\frac{1}{4x} = \frac{1}{x^3} \Rightarrow 4x = x^3 \Rightarrow x^3 - 4x = 0 \Rightarrow x(x^2 - 4) = 0 \Rightarrow x = 0, \pm 2$ \square

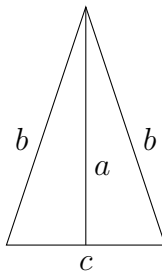
11. What is the probability that the product of the numbers rolled on three fair six-sided dice is prime?

- (A) $\frac{1}{36}$
- (B) $\frac{1}{24}$
- (C) $\frac{1}{16}$
- (D) $\frac{1}{12}$
- (E) $\frac{1}{8}$

SOLN One die must be prime and the others 1. There are 3 choices for the prime (2, 3, or 5), and 3 choices for which die is prime, so 9 ways this can happen. There are 6^3 outcomes possible, so the probability is $9/6^3 = \frac{1}{24}$. \square

12. The altitude a , equal sides b , and non-equal side c of an isosceles triangle have lengths that are, in the order listed, consecutive even numbers of centimeters. What is the area of the triangle?

- (A) 6 cm^2
- (B) 16 cm^2
- (C) 30 cm^2
- (D) 48 cm^2
- (E) 70 cm^2



SOLN Altitude $a = 2n$ for some integer n . The equal sides are $b = 2n + 2$, and the base is $c = 2n + 4$, so half of it is $n + 2$.

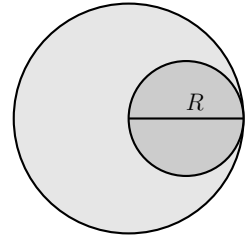
The altitude of an isosceles triangle splits it into two right triangles, so the Pythagorean Theorem gives

$$\begin{aligned} (n + 2)^2 + (2n)^2 &= (2n + 2)^2 \\ n^2 + 4n + 4 + 4n^2 &= 4n^2 + 8n + 4 \\ n^2 &= 4n \end{aligned}$$

If $n = 0$, there is no triangle. $n = 4$ gives an area of $\frac{1}{2}(12)(8) = 48 \text{ cm}^2$. \square

13. If a small circle's diameter is a large circle's radius, then the small circle's area is what percent of the large circle's area?

- (A) 20%
- (B) 25%
- (C) 40%
- (D) 50%
- (E) 55%



SOLN $A_{\text{small}} = \pi \left(\frac{R}{2}\right)^2$ $A_{\text{large}} = \pi R^2$
 so $A_{\text{small}} = \frac{1}{4}A_{\text{large}}$ $\frac{1}{4} = 25\%$ \square

14. Female bees are born from fertilized eggs and male bees are born from unfertilized eggs. This means that a male bee has only a mother, whereas a female has a mother and a father. If the ancestry of a male bee is traced back 8 generations, including the generation of the male bee, how many bees are there in all 8 generations?

- (A) 24
- (B) 42
- (C) 54
- (D) 61
- (E) 64

SOLN Each male has only a mother (female) and each female has both a male and female in the previous generation. Each generation only increases by the number of females.

gen					bees
1			m		1
2			f		1
3		f		m	2
4	f		m	f	3
5	f	m	f	f	5

Note: the number of bees in each generation is the Fibonacci sequence, so the answer is the sum of the first 8 terms of the Fibonacci sequence which is $1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 = 54$. \square

20. The substitute teacher for a fifth-grade class is trying to learn the names of the three girls (one of whom is named Debbie) and two boys in the front row. She knows they are in alphabetical order by last name, going from left to right: Benson, Harper, Kimball, Moss, Roker, and she can see they are boy, girl, boy, girl, girl, also from left to right. She also knows that Bobby's last name is Kimball, neither Jill nor Karen is wearing a headband (the middle girl is), and Karen is not sitting next to Brian. Who is sitting between the two boys?

- (A) Debbie
- (B) Karen
- (C) Jill
- (D) Brian
- (E) cannot be determined

SOLN Looking for the 1st girl on the left — you know it's not Brian, the middle girl has a headband and that is not Jill or Karen, it is Debbie. Karen isn't sitting by Brian so the girl between the two boys must be Jill. □