Snow College Jr. Mathematics Contest

key

March 18, 2025 **Junior Division: Grades 7–9** **Form: T**

Bubble in clearly the single best choice for each question you choose to answer.

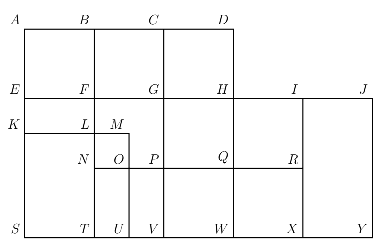
1. What is the sum of the prime factors of 2025?

   $\boxed{\text{(A)}}$ 22
   (B) 23
   (C) 25
   (D) 27
   (E) 28

   SOLN $2025 = 3^4 \cdot 5^2$; $3+3+3+3+5+5 = 22$ ▫

2. This diagram is filled with squares of various sizes. How many total squares are there?

   (A) 14
   (B) 15
   (C) 16
   $\boxed{\text{(D)}}$ 17
   (E) 18



   SOLN Begin with the smallest square $LMON$. Then count 9 squares that are double the length and width of the first. There is one that is 3 times the length and width of the first and 5 that are 4 times the length and width of the first. The last is the largest square $ADWS$ ▫

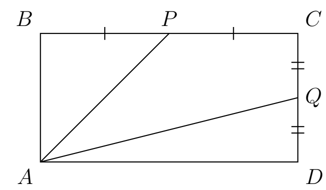
3. A person is jogging around a 1/4 mi track at 5 mph. If they jogged for 42 min, how many times did they go around the track?

   (A) 9
   (B) 12
   (C) 13
   $\boxed{\text{(D)}}$ 14
   (E) 15

   SOLN The time of 42 min is 42/60 hr. Multiply by 5 mi/hr to get 42/12 mi. Divide this by 1/4 or multiply by 4 gives $42/3 = 14$ laps. ▫

4. The area of quadrilateral $APCQ$ is $9\,\text{cm}^2$. What is the area of rectangle $ABCD$?

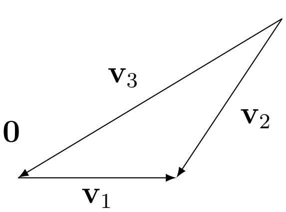
   (A) $12\,\text{cm}^2$
   (B) $14\,\text{cm}^2$
   (C) $15\,\text{cm}^2$
   $\boxed{\text{(D)}}$ $18\,\text{cm}^2$
   (E) $20\,\text{cm}^2$



   SOLN $\triangle ABP$ and $\triangle AQD$ both represent one quarter of the area of the rectangle since P and Q both bisect their sides. Thus $APCQ$ is half of the full rectangle which has an area of $18\,\text{cm}^2$. ▫

5. A vector is a quantity with both size and direction; they are often represented by arrows. Vectors $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ are shown below. Which of the following is the correct relationship between the three vectors?

   (A) $\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{v}_3$
   (B) $\mathbf{v}_1 - \mathbf{v}_2 = \mathbf{v}_3$
   (C) $\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 = \mathbf{0}$
   $\boxed{\text{(D)}}$ $\mathbf{v}_1 + \mathbf{v}_3 = \mathbf{v}_2$
   (E) $\mathbf{v}_3 - \mathbf{v}_2 = \mathbf{v}_1$



   SOLN Geometrically, vectors are added by placing a pair tip to tail as $\mathbf{v}_1$ and $\mathbf{v}_3$ are shown. The resultant vector is then drawn from the tail of the first to the tip of the second. ▫

6. If $2^x \cdot 3^y \cdot 7^z = 392$ and x, y, and z are integers, then what is xyz?

(A) -2

(B) 2

[(C)] 0

(D) 5

(E) 6

SOLN Prime factorization of 392 is $2^3 \cdot 7^2 \Rightarrow x = 3$, $y = 0$, and $z = 2$. □

7. Rationalize the denominator.

$$\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$$

[(A)] $\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}$

(B) $\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{12}$

(C) $\frac{6+\sqrt{10}-\sqrt{30}}{6}$

(D) $\frac{\sqrt{3}+3-\sqrt{15}}{6}$

(E) $\frac{2\sqrt{3}-3\sqrt{2}+\sqrt{30}}{4}$

SOLN We knock out the radicals in two steps. First multiply top and bottom by $(\sqrt{2}+\sqrt{3})-\sqrt{5}$.
$\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}} \cdot \frac{(\sqrt{2}+\sqrt{3})-\sqrt{5}}{(\sqrt{2}+\sqrt{3})-\sqrt{5}} = \frac{(\sqrt{2}+\sqrt{3})-\sqrt{5}}{(\sqrt{2}+\sqrt{3})^2-\sqrt{5}^2} = \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}$
Then multiply top and bottom by $\sqrt{6}$ to get $\frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}$ □

8. On a dark, cloudy night, Bud stands 15 ft away from a 20-ft tall light pole. The light on the pole is on and is the only source of light near Bud. Bud is 5 ft tall. What is the angle of elevation of the line segment joining the highest point of the light pole to the point on Bud's shadow that is farthest from the light pole?

(A) 20°

(B) 30°

(C) 40°

[(D)] 45°

(E) 60°

SOLN Use similar triangles to create the proportion equation $x/5 = (x+15)/20 \implies x = 5$. Since Bud and his shadow are both 5 ft the angle must be 45°. □

9. Replace each letter in ONE + ONE = TWO with a base-10 digit so that identical letters are replaced by identical digits and different letters are replaced with different digits, T is the only odd digit, and O cannot be zero. What is the value of N?

(A) 0

(B) 2

(C) 4

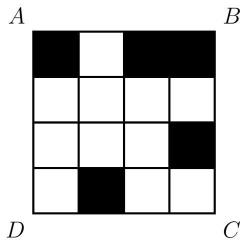
(D) 6

[(E)] 8

ONE	482
+ONE	+482
TWO	964

SOLN Considering the lead digits, $\mathsf{O} \leq 4$.
E is 0, 2, 4, 6, or 8.
If E were 0 then O would also be 0.
If E were 6 or 8 then there would be a carry to the tens place, contradicting that W is even.
If E were 4 then O would be 8, contradicting that $\mathsf{O} \leq 4$.
Thus $\mathsf{E} = 2$ and $\mathsf{O} = 4$.
N is 0, 6, or 8.
If N were 0 then W would also be 0.
If N were 6 then W would be 2, contradicting that $\mathsf{E} = 2$.
Thus $\mathsf{N} = 8$. □

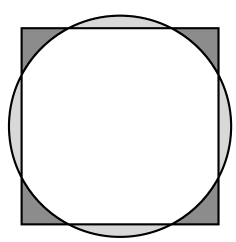
10. What is the minimum number of small squares that must be colored black so that a line of symmetry lies on the diagonal $\overline{BD}$ of square $ABCD$?



(A) 1
(B) 2
(C) 3
[(D)] 4
(E) 5

SOLN The black square in the upper right corner is the only one that doesn't need a new mate. □

11. A square with side length 2 and a circle share the same center. The total area of the regions that are inside the circle and outside the square is equal to the total area of the regions that are outside the circle and inside the square. What is the circle's radius?



[(A)] $\frac{2}{\sqrt{\pi}}$
(B) $\frac{1+\sqrt{2}}{2}$
(C) $\frac{3}{2}$
(D) 4
(E) 5

SOLN Soln #1: Let the region within the circle **and** the square be a and the radius r. The area of the circle minus a is equal to the area of the square minus a.

$$\pi r^2 - a = 4 - a$$

$$r^2 = \frac{4}{\pi}$$

Soln #2: Since the areas of the regions outside of the circle and square are equal, the area of the circle must equal the area of the square: $\pi r^2 = 4$ □

12. Alice and Bob play a game involving a circle whose circumference is divided by 12 equally-spaced points. The points are numbered clockwise from 1 to 12. Both start on point 12. Alice moves clockwise around the circle and Bob counterclockwise. In each turn of the game, Alice moves 5 points clockwise and Bob moves 9 points counterclockwise. The game ends when they stop on the same point. How many turns will this take?

[(A)] 6
(B) 8
(C) 12
(D) 14
(E) 24

SOLN Alice moves $5k$ steps and Bob moves $9k$ steps, where k is the turn they are on. Alice and Bob coincide when the number of steps they move collectively, $14k$, is a multiple of 12. If $14k$ is a multiple of 12 and 14 has a factor of 2 then k must have a factor of 6. The smallest number that is a multiple of 6 is 6. □

13. The Little Twelve Basketball Conference has two divisions, with six teams in each division. Each team plays each of the other teams in its own division twice and every team in the other division once. How many conference games are scheduled?

(A) 80
[(B)] 96
(C) 100
(D) 108
(E) 124

[SOLN] Soln #1: Within each division there are $\binom{6}{2} = 15$ pairings and each of these games happens twice. The same goes for the other division, so there are $4(15) = 60$ games within their own divisions. The number of games between the two divisions is $(6)(6) = 36$. The total is $60 + 36 = 96$.
Soln #2: Each team plays 10 games in its own division and 6 games against the other division. Each of the 12 teams plays 16 games. Because each game involves two teams the total is $\frac{12\times16}{2}$. □

14. Each day, Jenny ate 20% of the jellybeans that were in her jar at the beginning of that day. At the end of the second day, 32 remained. How many jellybeans were in the jar originally?

(A) 40
[(B)] 50
(C) 55
(D) 60
(E) 75

[SOLN] Let x be the starting number.
$0.8^2 \cdot x = 32 \Rightarrow x = 32/0.64 = 1/0.02$ □

15. For the dataset $\{15, 14, 15, 5, 12, 5\}$, compute the value of **mean** − **median**.

(A) 1
[(B)] −2
(C) 2
(D) −1
(E) −0.5

[SOLN] The data sum to 66 so the mean is 11 while the median is $(14 + 12)/2 = 13$ for a difference of -2. □

16. It is 2025 and Danni is not old enough to vote but has had her birthday this year. Danni noticed that her age is equal to the sum of the digits in her birth year. What is the sum of the digits in her birth year?

(A) 7
[(B)] 9
(C) 12
(D) 14
(E) 15

[SOLN] One could try a brute force method, checking each year until the right age is found. Alternatively, let the birth year be $20xy$. Sum of digits of birth year $= 2 + 0 + x + y$. Age $= 2025 - (2000 + 10x + y) = 25 - 10x - y$. Setting this equal to the sum of digits and arithmetic yield $23 = 11x + 2y$. Thus $x = 1,\ y = 6$, so Age $= 9$ and sum of digits in birth year is $2 + 0 + 1 + 6 = 9$. □

17. What is the area of the largest rectangle that can be inscribed in a closed semicircle of radius 4?

(A) 32

(B) 16

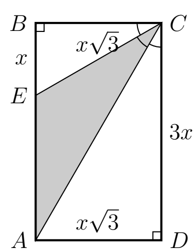
(C) $8\sqrt{2}$

(D) $16\sqrt{2}$

(E) $12\sqrt{3}$

SOLN Make a complete circle and notice the largest area rectangle inscribed in the circle is a square of side length $4\sqrt{2}$. This square has area 32 so the semicircle rectangle will have area 16. □

18. $ABCD$ is a rectangle. The three measured angles at vertex C are all congruent. What fraction of the rectangle is shaded?



(A) $\sqrt{3}/4$

(B) $1/3$

(C) $1/2$

(D) $\sqrt{3}/5$

(E) $1/4$

SOLN The three congruent angles at C are all 30°. Each of the non-shaded triangles are $30° - 60° - 90°$ triangles. Label point E and segment BE as a length of x. Then BC and AD measure $x\sqrt{3}$, and CD measures $3x$. From this we get EA measures $2x$ and the shaded area out of the total area will be

$$\frac{(1/2)(2x)(x\sqrt{3})}{(3x)(x\sqrt{3})} = \frac{1}{3}$$ □

19. There are 396 men, women, and children in a room. If the ratio of women to men is 2:3, and the ratio of men to children is 1:2, how many men are in the room?

(A) 79

(B) 82

(C) 86

(D) 95

(E) 108

SOLN

$$m + w + c = 396$$

$$\frac{w}{m} = \frac{2}{3} \implies w = \frac{2}{3}m$$

$$\frac{m}{c} = \frac{1}{2} \implies c = 2m$$

$$m + \frac{2}{3}m + 2m = 396$$

$$\frac{11}{3}m = 396 \implies m = 108$$ □

20. For what value of k is the line through the points $(3, 2k + 1)$ and $(8, 4k - 5)$ parallel to the x-axis?

(A) -4

(B) 2

(C) 0

(D) -1

(E) 3

SOLN The two points must have the same y-value. $2k + 1 = 4k - 5 \implies k = 3$ □