Snow College Mathematics Contest

March 18, 2025 **Senior Division: Grades 10-12** **Form: T**

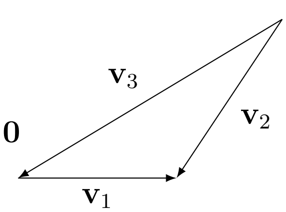
Bubble in clearly the single best choice for each question you choose to answer.

1. Kim plays basketball for her school. Her free-throw shooting percentage for the season was 75% exactly before today. During tonight's game she makes all five free throws, bringing her percentage up to 80%. How many free throws has Kim made in the season (including tonight)?

   (A) 20
   (B) 22
   (C) 24
   (D) 25
   (E) 28

2. A vector is a quantity with both size and direction; they are often represented by arrows. Vectors $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ are shown below. Which of the following is the correct relationship between the three vectors?

   (A) $\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{v}_3$
   (B) $\mathbf{v}_1 - \mathbf{v}_2 = \mathbf{v}_3$
   (C) $\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 = \mathbf{0}$
   (D) $\mathbf{v}_3 - \mathbf{v}_2 = \mathbf{v}_1$
   (E) $\mathbf{v}_1 + \mathbf{v}_3 = \mathbf{v}_2$



3. Professor Spinner is playing basketball with her son, Fidget. Fidget is fouled and gets two foul shots. The probability that Fidget makes the first shot is $\frac{3}{4}$. If he makes the first shot, the probability that he makes the second shot is $\frac{9}{10}$. If he misses the first shot, the probability that he makes the second shot is $\frac{1}{2}$. What is the probability that Fidget makes at least one shot?

   (A) $\frac{3}{40}$
   (B) $\frac{1}{8}$
   (C) $\frac{5}{8}$
   (D) $\frac{27}{40}$
   (E) $\frac{7}{8}$

4. Mr. Pierce has 10 students show up for his class service project on Earth Day. He needs to assign 7 of the students to plant trees around the school. The remaining 3 students will plant trees at the public library. How many different ways can Mr. Pierce assign his 10 students to these two tree-planting locations?

   (A) 120
   (B) 240
   (C) 720
   (D) 13400
   (E) 604800

5. If a k^{th} power *generalized mean* is defined as

$$M_k = \left(\frac{a^k + b^k}{2}\right)^{1/k}$$

we get the RMS (root-mean-square) value for $k = 2$, the geometric mean for $k \to 0$, and the harmonic mean for $k = -1$. Which value of k produces the arithmetic mean?

(A) -2

(B) 0.5

(C) 1

(D) 3

(E) 4

6. Consider the points $A(-5, 1)$ and $C(3, -3)$ shown below. A point $B(b, 0)$ is placed along the x-axis. The shortest distance from A to C through B occurs if B is on the line containing A and C. But what value of b will give the minimum of $|AB|^2 + |BC|^2$?

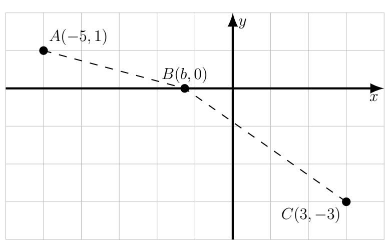
(A) -5

(B) -3

(C) -2

(D) -1

(E) 0



7. Find the total area between $y = \sin(x)$ and $y = \cos(x)$ on the interval $[0, \pi]$.

(A) $2\sqrt{2}$

(B) 2

(C) $5\pi/6$

(D) $\pi - 1/4$

(E) -2

8. One extension of the real numbers is the complex numbers $z = a + b\mathrm{i}$, where a, b are real numbers and $\mathrm{i}^2 = -1$. ($\bar{z} = a - b\mathrm{i}$.) Another is the dual numbers $z = a + b\varepsilon$, where a, b are real numbers and $\varepsilon^2 = 0$. A "unit circle" is the locus of points z such that $|z| = 1$, or, equivalently, $z\bar{z} = 1$; in the complex plane this indeed looks like a circle. What is the "unit circle" in the dual number plane?

(A) All dual numbers where $a + b = 1$

(B) All dual numbers where $b = \pm 1$

(C) All dual numbers where $a^2 = b^2$

(D) All dual numbers where $a = \pm 1$

(E) All dual numbers where $ab = 1$

9. The figure shows only the first six of a sequence of squares. The outermost square is a unit square with area $1\,\text{cm}^2$. Each of the other squares is obtained by joining the midpoints of the sides of the squares before it. Find the sum of the infinite series of the perimeters (in cm) of all the squares.

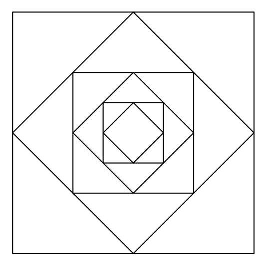
(A) $4\sqrt{2}$

(B) $2(2 - \sqrt{2})$

(C) $8/(2 - \sqrt{2})$

(D) 4

(E) 8



10. Suppose that a and b are real numbers and f is differentiable at x. Express the limit

$$\lim_{h \to 0} \frac{f(ah + x) - f(bh + x)}{h}$$

in terms of a, b, and $f'(x)$.

(A) $\frac{1}{a} - \frac{1}{b}$

(B) $\frac{f'(x)}{a} - \frac{f'(x)}{b}$

(C) $f'(ax) + f'(bx)$

(D) $f'(ax) - f'(bx)$

(E) $af'(x) - bf'(x)$

11. For which x value (other than 0) does the graph of $y = x^2 e^{-x}$ have a tangent line with a y-intercept of 0?

(A) 1

(B) 2

(C) $e - 1$

(D) e

(E) $e + 1$

12. Solve the equation for x.

$$x + \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}} = 9$$

(A) $x = 3$

(B) $x = 4$

(C) $x = 5$

(D) $x = 6$

(E) $x = 7$

13. Solve for s.

$$\sqrt{s\sqrt{n\sqrt{o\sqrt{w}}}} = 2$$

$$s > 0, \quad s = n = o = w$$

(A) $s = 2\sqrt[15]{2}$

(B) $s = \sqrt[16]{2}$

(C) $s = 2^{16}\sqrt{2}$

(D) $s = 2^8$

(E) $s = 2\sqrt[16]{2}$

14. Which is **NOT** equal to 2025?

(A) $1^3 + 2^3 + 3^3 + \ldots + 9^3$

(B) 45^2

(C) base two (binary): 11111101001

(D) $1^2 + 2^2 + 3^2 \ldots + 18^2$

(E) base five: 31100

15. Suppose that Ali has three investment opportunities, where she magically knows the future performance:

- Option A: gain 10% in year 1, lose 10% in year 2, gain 10% in year 3.
- Option B: gain 5% in year 1, gain 4% in year 2, no change in year 3.
- Option C: gain 20% in year 1, lose 5% in year 2, lose 5% in year 3.

If Ali wants to end up with the most money, which investment should she choose?

(A) Option A only

(B) Option B only

(C) Option C only

(D) Option A or C

(E) All options are equal.

16. Suppose that f is a function with domain $D = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$. Suppose that the outputs of the function $f(x)$ are *chosen* from the set $C = \{7, 8, 9\}$. Suppose that A is a subset of C. Define $f^{-1}(A)$, called the *preimage* of A, to be the set $\{x\ :\ f(x) \in A\}$. If $f^{-1}(\{7\}) = \{1,\ 2\}$ and $f^{-1}(\{8\}) = \{3,\ 5\}$, then compute:

$$f^{-1}(\{f(4)\})$$

(A) $\{4\}$

(B) $\{1, 2\}$

(C) $\{3, 5\}$

(D) $\{6\}$

(E) $\{4, 6\}$

17. What is the area of the largest rectangle that can be inscribed in a closed semicircle of radius 4?

(A) 32

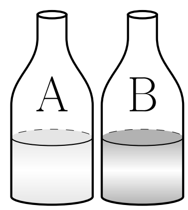
(B) 16

(C) $8\sqrt{2}$

(D) $16\sqrt{2}$

(E) $12\sqrt{3}$

18. Jug A has 1 L of lemonade and jug B has 1 L of fruit punch. Pour x mL of the lemonade from A into B, mix well, and then pour x mL of the mixture from B back into A so each jug again contains 1 L of beverage. Will the amount of fruit punch in A be more than, less than, or equal to the amount of lemonade in B?



(A) more than

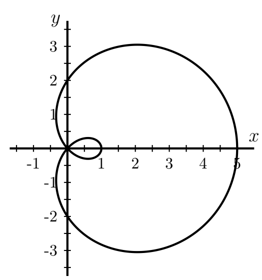
(B) less than

(C) equal to

(D) depends on x

(E) not enough information

19. Which polar equation best represents the graph for $0 \le \theta \le 2\pi$?



(A) $r = 3 - 2\sin\theta$

(B) $r = 2 + 3\sin\theta$

(C) $r = 3 + 2\sin\theta$

(D) $r = 3 + 2\cos\theta$

(E) $r = 2 + 3\cos\theta$

20. Which pair of cubic *weird dice* produces the same probability distribution of sums as a pair of regular dice?

(A) 1, 2, 2, 3, 3, 4 and 1, 3, 4, 5, 6, 8

(B) 1, 1, 3, 3, 3, 5 and 1, 2, 4, 6, 6, 7

(C) 1, 2, 2, 3, 3, 3 and 2, 3, 4, 5, 5, 9

(D) 0, 2, 2, 4, 4, 4 and 2, 3, 4, 5, 6, 7

(E) 1, 2, 2, 3, 4, 4 and 2, 3, 4, 5, 6, 8

21. What is the output of the following Python program?

```
(a, b) = (1, 1)
while a < 20:
  print(a)
  (a, b) = (b, a + b)
```

(A) the Fibonacci numbers less than 20

(B) the squares of numbers less than 20

(C) the triangular numbers less than 20

(D) the counting numbers less than 20

(E) the prime numbers less than 20

22. For what value of k is the line through the points $(3, 2k+1)$ and $(8, 4k-5)$ parallel to the x-axis?

(A) -4

(B) 2

(C) 0

(D) -1

(E) 3

23. A highly composite number is a positive integer with more divisors (factors) than any other positive integer less than it. For example, 6 is highly composite because it has four divisors (1, 2, 3, 6) and no positive integer less than 6 has that many. What is the next highly composite number?

(A) 8

(B) 9

(C) 10

(D) 12

(E) 24

24. A factorial prime is a prime number that is one less or one more than a factorial. Which of the following is a factorial prime?

(A) 11

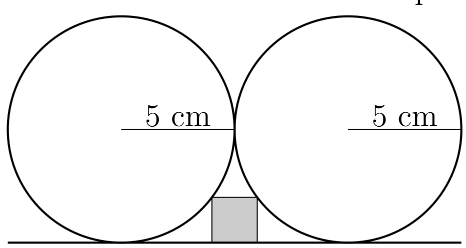
(B) 15

(C) 19

(D) 23

(E) 119

25. Two circles of radius 5 cm are tangent to the same line and tangent to each other. A square is inscribed between the circles and the line. What is the area of the square?



(A) $2\,\text{cm}^2$

(B) $3\,\text{cm}^2$

(C) $4\,\text{cm}^2$

(D) $4.25\,\text{cm}^2$

(E) $5\,\text{cm}^2$

26. Find the solution to the following equation.

$$\left(\frac{1}{4}\right)^{x-4} = (\sqrt{2})^{6x-4}$$

(A) 5/2

(B) 2

(C) 3

(D) 1/2

(E) 5

27. If the values of a, b, and c are real, the solutions of $ax^2 + bx + c = 0$ may be rational, irrational, or complex ($\text{i} = \sqrt{-1}$). In general, if a, b, and c are complex, then the solutions will be complex. Find the solutions to the following quadratic equation.

$$x^2 + \text{i}x + 2 = 0$$

(A) $\pm\text{i}$

(B) $\text{i}, -2\text{i}$

(C) $2\text{i}, -\text{i}$

(D) $\pm 2\text{i}$

(E) $\text{i}, 2\text{i}$

28. Simplify the expression.

$$\sin\left[\tan^{-1}\left(\frac{3}{4}\right) + \cos^{-1}\left(\frac{5}{13}\right)\right]$$

(A) 8/17

(B) 43/65

(C) 15/52

(D) 17/52

(E) 63/65

29. Solve the equation for n.

$$\frac{n}{(n-1)!} - \frac{13}{n!} = \frac{n-1}{n!}$$

(A) 2

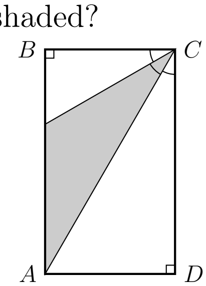
(B) 3

(C) 4

(D) 5

(E) 24

30. $ABCD$ is a rectangle. The three measured angles at vertex C are all congruent. What fraction of the rectangle is shaded?



(A) $\sqrt{3}/4$

(B) 1/3

(C) 1/2

(D) $\sqrt{3}/5$

(E) 1/4

31. The general product of 2×2 matrices is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}.$$

The Hadamard (or Schur) product of matrices is found by multiplying components element-wise as illustrated below.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae & bf \\ cg & dh \end{bmatrix}$$

Which of the following is NOT a necessary condition for the general product to be equal to the Hadamard product?

(A) $bg = 0$

(B) $g = \frac{ce}{d-c}$

(C) $ce + dg = cg$

(D) $cf = 0$

(E) $f = \frac{bh}{b-a}$

32. There are 396 men, women, and children in a room. If the ratio of women to men is 2:3, and the ratio of men to children is 1:2, how many men are in the room?

(A) 79

(B) 82

(C) 86

(D) 95

(E) 108

33. Mick's current age is double what Ruth's age was 35 years ago. 20 years ago, Mick's age was half of Ruth's current age. What is the difference between the current ages of Ruth and Mick?

(A) 5

(B) 10

(C) 15

(D) 20

(E) 24

34. If $A = 1001011_{\text{two}}$, $B = 33_{\text{four}}$, and $C = 12_{\text{eight}}$, what is $A \div B \times C$ in base ten?

(A) 50

(B) 364004

(C) 0.5

(D) 14

(E) 35

35. In the Fibonacci sequence each number is the sum of the two preceding numbers, with the first two both 1.
The sum of the first and third of three consecutive Fibonacci numbers is three more than twice the second. Three times the first is eleven more than the second. What is the sum of the three numbers?

(A) 16

(B) 26

(C) 42

(D) 68

(E) 110

36. Napoleon needs to load Tina, his 300-pound pet llama, into a truck to take her to the vet for a checkup. The ramp to the truck has an angle of elevation of 30°. How much force is required to pull Tina up the frictionless ramp leading into the truck?

(A) 130 pounds

(B) 150 pounds

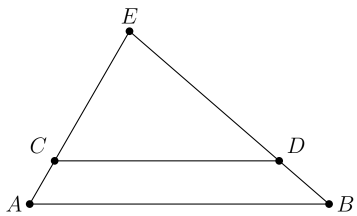
(C) 173 pounds

(D) 200 pounds

(E) 260 pounds

37. $\overline{AB}$ is parallel to $\overline{CD}$.
$AC = 6\,\text{ft}$, $CE = 18\,\text{ft}$, and $CD = 27\,\text{ft}$.
What is the length of $\overline{AB}$?

(A) 9 ft
(B) 20 ft
(C) 33 ft
(D) 36 ft
(E) 54 ft



38. It is 2025 and Dan is an adult who has had his birthday this year. Dan noticed that his age is equal to the sum of the digits in his birth year. What is the sum of the digits in his age?

(A) 7
(B) 9
(C) 12
(D) 14
(E) 15

39. Let M be the **sum of the solutions** to $e^{-x}\sin x - e^{-x}\cos x = 0$ in $[0, 2\pi]$.
Find $\csc M$.

(A) -1
(B) $-\frac{\sqrt{3}}{2}$
(C) 1
(D) $\frac{2}{\sqrt{3}}$
(E) 2

40. Solve. $\log_5(\log_2 x) = 1$

(A) 1
(B) 7
(C) 10
(D) 25
(E) 32