Bubble in the single best choice for each question you choose to answer.

1. When rolling two fair dice, what is the probability of getting a sum of 7?

   (A) $\frac{1}{7}$

   (B) $\frac{1}{36}$

   (C) $\frac{1}{6}$

   (D) $\frac{7}{36}$

   (E) $\frac{1}{11}$

   SOLN There are 36 total possible outcomes from rolling two fair dice; six of them have a sum of 7.

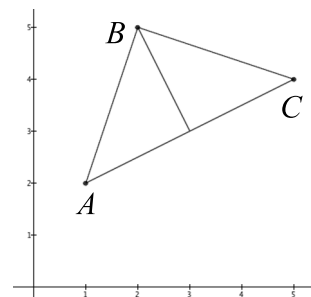 | | 1 | 2 | 3 | 4 | 5 | 6 |
   |---|---|---|---|---|---|---|
   | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
   | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
   | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
   | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
   | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

2. The supplement of an angle is $2\frac{1}{2}$ times the complement of the angle. Find the measure of the angle.

   (A) 20°

   (B) 30°

   (C) 40°

   (D) 50°

   (E) 60°

   SOLN $(180 - x) = \frac{5}{2}(90 - x) \implies$
   $180 - x = 225 - \frac{5}{2}x \implies \frac{3}{2}x = 45$

3. What is the area enclosed by the triangle whose vertices are the points $A(1,2)$, $B(2,5)$, $C(5,4)$?

   (A) $\sqrt{5}$

   (B) 3

   (C) 5

   (D) 6

   (E) 9

   SOLN If we use AC as the base of the triangle we get $b = \sqrt{20}$ and $h = \sqrt{5}$. $A = \frac{1}{2}bh = \frac{1}{2}(2\sqrt{5})(\sqrt{5})$. But using AB or AC as the base (and the other as the height, since it is a right triangle) is easier: $A = \frac{1}{2}bh = \frac{1}{2}(\sqrt{10})(\sqrt{10})$. Or by determinant the area is:
   $$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 5 & 4 & 1 \\ 2 & 5 & 1 \end{vmatrix} = 5$$

4. Simplify $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$.

   (A) 1

   (B) 3

   (C) $\sqrt{13}$

   (D) 12

   (E) 21

   SOLN $(a + b)(a - b) = a^2 - b^2$, so $\sqrt{5}^2 - \sqrt{2}^2 = 5 - 2$

5. Which of the following is an equation for the line through $(3, 2)$ which is perpendicular to the line $3x - 4y = 7$?

(A) $3x - 4y = 1$

(B) $3x + 4y = 17$

(C) $4x - 3y = 6$

(D) $4x + 3y = 18$

(E) $4x + 3y = 6$

$\boxed{\mathscr{SOLN}}$ The slopes of perpendicular lines are negative reciprocals of each other. The slope of the given line is $m = \frac{3}{4}$. Only D and E have a slope of $-\frac{4}{3}$, and only D goes through $(3, 2)$. □

6. If $4x$ is the reciprocal of $1/x^3$ then which could be x?

(A) $\frac{1}{8}$

(B) $\frac{1}{2}$

(C) 2

(D) 8

(E) 4

$\boxed{\mathscr{SOLN}}$ $4x = \frac{1}{(1/x^3)} = x^3 \implies 4 = x^2$ □

7. When fully expanded $10\,000^{9999}$ has how many digits?

(A) 9999

(B) $10\,000$

(C) $19\,999$

(D) $39\,996$

(E) $39\,997$

$\boxed{\mathscr{SOLN}}$ $10\,000^{9999} = (10^4)^{9999} = 10^{39\,996}$
That's 1 followed by 39 996 zeroes. □

8. If a recipe calls for $2\frac{3}{4}$ c flour to make 3 dozen cookies, how much flour is required to make 7 dozen cookies?

(A) $4\frac{7}{12}$ c

(B) $5\frac{1}{2}$ c

(C) $6\frac{5}{12}$ c

(D) $7\frac{7}{4}$ c

(E) $19\frac{1}{4}$ c

$\boxed{\mathscr{SOLN}}$ Use ratios: $\frac{2\frac{3}{4}\,\text{c}}{3\,\text{doz.}} = \frac{x\,\text{c}}{7\,\text{doz.}}$ □

9. Sue took her collection of nickels and dimes to deposit in the bank. She has five fewer nickels than dimes. Her total deposit was $27.05. How many dimes did she deposit?

(A) 177

(B) 182

(C) 187

(D) 225

(E) 359

$\boxed{\mathscr{SOLN}}$ Let x be the number of dimes; then $x - 5$ is the number of nickels. The value of the dimes is $0.10x$ and the nickels $0.05(x - 5)$.
$0.10x + 0.05(x - 5) = 27.05$
$0.15x - 0.25 = 27.05$ □

10. A stack of firewood has 35 pieces on the bottom row, 31 pieces on top of those, then 27 pieces, and so on. If there are 150 pieces of wood total, how many rows are there?

(A) 5

(B) $6 = |\{35, 31, 27, 23, 19, 15\}|$

(C) 7

(D) 8

(E) 15

$\boxed{\mathscr{SOLN}}$ $150 - 35 - 31 - 27 - 23 - 19 - 15 = 0$
or $150 = 35 + 31 + 27 + 23 + 19 + 15$ □

11. Powers of two are additive building blocks of the whole numbers; that is, each whole number can be expressed as the sum of powers of two (with all different powers) in a unique way. For example, $10 = 2^3 + 2^1$. What is the sum of the exponents in such an expression for 127?

(A) 11

(B) 13

(C) 15

(D) 18

(E) $21 = 6 + 5 + 4 + 3 + 2 + 1 + 0$

$\boxed{\mathscr{SOLN}}$ $2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 =$
$127 = 64 + 32 + 16 + 8 + 4 + 2 + 1$ □

13. Cal, Hal, and Sal cooked beans for dinner. Cal contributed 400 grams of beans and Hal contributed 200 grams of beans. Sal did not have any beans, so she contributed \$6. What is the fairest way to divide the \$6 between Cal and Hal? Assume the three equally shared the dinner.

(A) Cal: \$4, Hal: \$2

(B) Cal: \$0, Hal: \$6

(C) Cal: \$2, Hal: \$4

$\boxed{(D)}$ Cal: \$6, Hal: \$0

(E) Cal: \$3, Hal: \$3

$\boxed{\mathscr{SOLN}}$ Each person ate 200 g of beans. Therefore, Cal should receive the entire \$6 since he supplied the entire 200 g of beans eaten by Sal. Hal simply ate what he brought. □

12. Which of the following fractions lies on the number line between $\frac{15}{34}$ and $\frac{16}{31}$?

(A) $\frac{9}{17}$

(B) $\frac{37}{101}$

(C) $\frac{13}{25}$

$\boxed{(D)}$ $\frac{31}{65}$

(E) $\frac{14}{27}$

$\boxed{\mathscr{SOLN}}$ $\dfrac{a}{b} < \dfrac{a+c}{b+d} < \dfrac{c}{d}$

if $c, d > 0$ and $\frac{c}{d} > \frac{a}{b}$. □

14. The harmonic mean m of two numbers a and b is given by

$$m = \frac{2}{\left(\frac{1}{a} + \frac{1}{b}\right)}$$
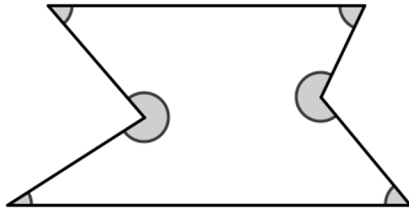
What is the harmonic mean of 3 and 6?

(A) 1

(B) 3

(C) $\frac{9}{2}$

(D) $\frac{16}{3}$

$\boxed{(E)}$ 4

$\boxed{\mathscr{SOLN}}$ $m = \dfrac{2}{\left(\frac{1}{3} + \frac{1}{6}\right)} = \dfrac{2}{\frac{1}{2}}$ □

15. If the top and bottom segments are parallel, what is the sum of the measures of all the interior angles in the concave hexagon?

    (A) 540°

    (B) 720°

    (C) 800°

    (D) 900°

    (E) 1080°

    soln The sum of the interior angles of an n-gon is $(n-2) \times 180°$. Here the sum is $(6-2) \times 180°$. That the top and bottom are parallel is irrelevant, as is the concave nature of the hexagon. □

16. In a group of 22 students, 12 like to play basketball, 15 like to play soccer, but two don't like to play either sport. How many like to play both basketball and soccer?

    (A) 7

    (B) 27

    (C) 3

    (D) 5

    (E) 9

    Venn Diagram
    Basketball   Soccer
    $12 - x$ x $15 - x$
    2
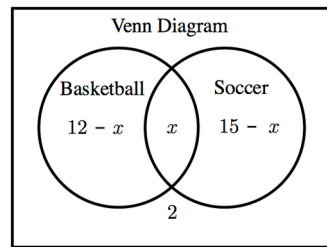
    soln $(12-x)+x+(15-x)+2 = 22 \implies 29 - x = 22 \implies x = 7$ □

17. The number represented as 256 in base 10 has what base 5 representation?

    (A) 128

    (B) 211

    (C) 310

    (D) 512

    (E) 2011

    soln $256_{10} = 2 \cdot 5^3 + 0 \cdot 5^2 + 1 \cdot 5^1 + 1 \cdot 5^0 = 2011_5$ □

18. What is the x-coordinate of the x-intercept?
    $$5y = 3x - 20$$

    (A) -20

    (B) -4

    (C) $\frac{3}{5}$

    (D) $\frac{5}{3}$

    (E) $\frac{20}{3}$

    soln $0 = 3x - 20 \implies 20 = 3x$. □

19. Following the pattern, how many consecutive odd integers starting with 1 are required to sum to 169?
    $$1 + 3 = 4$$
    $$1 + 3 + 5 = 9$$
    $$1 + 3 + 5 + 7 = 16$$
    $$1 + 3 + 5 + 7 + 9 = 25$$

    (A) 11

    (B) 13

    (C) 17

    (D) 19

    (E) 21

    soln The sums are n^2 where n is the number of integers being added. □

20. Ralph's company was losing money. As a result Ralph received a 25% pay cut. By what percentage must his new salary be raised to bring it back to the original level?

    (A) 25%

    (B) $33\frac{1}{3}\%$

    (C) 40%

    (D) 50%

    (E) 100%

    soln Let x be Ralph's old pay rate and y his new rate. $y = \frac{3}{4}x$ so $x = \frac{4}{3}y$ □