

Snow College Mathematics Contest

April 2, 2013

Senior Division: Grades 10-12

Form: **T**

Bubble in the single best choice for each question you choose to answer.

1. When rolling two fair dice, what is the probability of getting a sum of 7?

- (A) $\frac{1}{7}$
- (B) $\frac{1}{36}$
- (C) $\frac{1}{6}$
- (D) $\frac{7}{36}$
- (E) $\frac{6}{7}$

2. Find the sum: $\frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^4 + \dots$

- (A) $\frac{4}{3}$
- (B) $\frac{1}{3}$
- (C) $\frac{5}{4}$
- (D) $\frac{1}{2}$
- (E) ∞

3. The Pauli spin matrices σ_1 , σ_2 , and σ_3 appear in quantum mechanics. They are

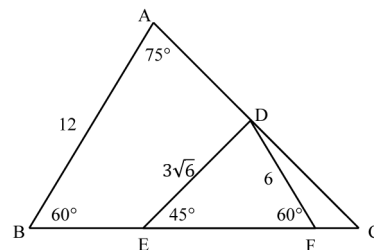
$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The *trace* of a matrix is the sum of the elements on the main diagonal: $\text{Tr}(A) = \sum_{i=1}^n a_{ii}$. What is $\text{Tr}(\sigma_1 + \sigma_2 + \sigma_3)$?

- (A) 0
- (B) -1
- (C) i
- (D) -i
- (E) I

4. Referring to the figure, find AC.

- (A) $4\sqrt{6}$
- (B) $6\sqrt{6}$
- (C) $8\sqrt{3}$
- (D) $8\sqrt{6}$
- (E) $12\sqrt{2}$



5. Any point on the earth makes one revolution about the earth's axis in 24 h. If the radius of the earth is 3950 mi, what is the linear velocity, in miles per hour, of a point on the equator? Use approximations.

- (A) 42 mph
- (B) 165 mph
- (C) 392 mph
- (D) 518 mph
- (E) 1034 mph

6. Some students rented a boat during spring break. They went 60mph from the dock to an island 60 miles away. They immediately turned around and returned at a more leisurely 30mph. What was their average speed for the whole trip?

- (A) 50 mph
- (B) 48 mph
- (C) 45 mph
- (D) 42 mph
- (E) 40 mph

7. What is the output of the following BASIC computer program with some natural number n given as input?

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10 input n
20 f = 1
30 for i = 1 to n
40 f = f * i
50 next i
60 print f

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- (A) $n!$
 (B) the prime factors of n
 (C) least common multiple of n and f
 (D) n th triangular number
 (E) n^n

8. What is the remainder when $x^3 - 2x^2 + 4$ is divided by $x + 2$?

- (A) -12
 (B) 0
 (C) 4
 (D) 6
 (E) 12

9. Powers of two are additive building blocks of the whole numbers; that is, each whole number can be expressed as the sum of powers of two (with all different powers) in a unique way. For example, $10 = 2^3 + 2^1$. What is the sum of the exponents in such an expression for 127?

- (A) 11
 (B) 13
 (C) 15
 (D) 18
 (E) 21

10. Two black balls and one white ball (identical except for color) are in a bag. A ball is drawn, its color recorded, and then the ball is replaced in the bag and the balls are mixed up. This process is done four times. What is the probability that of the four balls drawn that exactly two of them are black?

- (A) $\frac{4}{81}$
 (B) $\frac{1}{2}$
 (C) $\frac{8}{27}$
 (D) $\frac{8}{81}$
 (E) $\frac{3}{8}$

11. Which is a solution of the equation?

$$\cos^2 \theta = \frac{2 + \sqrt{3}}{4}$$

- (A) $\frac{\pi}{18}$
 (B) $\frac{\pi}{12}$
 (C) $\frac{\pi}{9}$
 (D) $\frac{\pi}{6}$
 (E) $\frac{\pi}{3}$

12. Characterize the roots of $x^3 + 6x^2 + 11x + 6$.

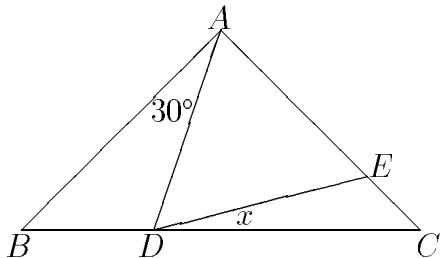
- (A) no negative roots
 (B) no positive roots
 (C) no real roots
 (D) 1 positive root, 2 negative roots
 (E) 1 negative root, 2 positive roots

13. If 2 is a solution of $x^3 + hx + 10 = 0$, then what is h ?

- (A) 10
 (B) 9
 (C) 2
 (D) -2
 (E) -9

14. In the figure $\overline{AB} = \overline{AC}$, $\angle BAD = 30^\circ$, and $\overline{AE} = \overline{AD}$. Find the measure of angle x .

- (A) $7\frac{1}{2}^\circ$
 (B) 10°
 (C) $12\frac{1}{2}^\circ$
 (D) 15°
 (E) 20°



15. If a recipe calls for $2\frac{3}{4}$ c flour to make 3 dozen cookies, how much flour is required to make 7 dozen cookies?

- (A) $4\frac{7}{12}$ c
 (B) $5\frac{1}{2}$ c
 (C) $6\frac{5}{12}$ c
 (D) $7\frac{7}{4}$ c
 (E) $19\frac{1}{4}$ c

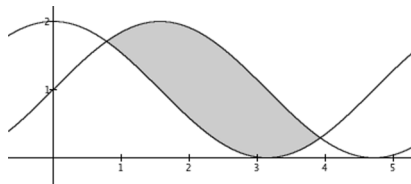
16. Find the solution set.

$$\log_5(x + 6) + \log_5(x - 6) = 3$$

- (A) $\{\sqrt{161}\}$
 (B) $\{\frac{125}{3}\}$
 (C) $\{\frac{183}{5}\}$
 (D) $\{161\}$
 (E) $\{-\sqrt{161}, \sqrt{161}\}$

17. What is the area between $f(x) = \sin x + 1$ and $g(x) = \cos x + 1$ between any two successive crossings?

- (A) π
 (B) $\pi/2$
 (C) $3\pi/2$
 (D) $2\pi/3$
 (E) $2\sqrt{2}$

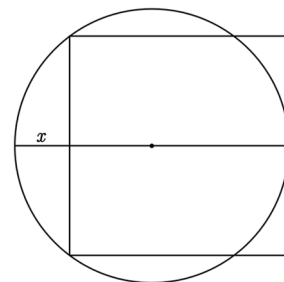


18. a modulo b means the remainder when a is divided by b ; for example, 14 modulo 3 = 2. The set $\mathbb{F} = \{0, 2, 4, 6, 8\}$ together with the binary operations of addition and multiplication modulo 10 is a *field*. Which is the unity element (multiplicative identity)?

- (A) 0
 (B) 2
 (C) 4
 (D) 6
 (E) 8

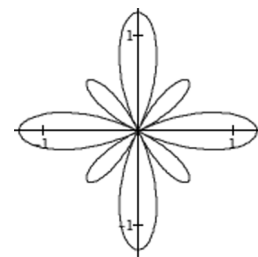
19. A square of side 8 meets a circle tangent to a side with opposite corners incident with the circle. Find x along the circle's diameter.

- (A) 2
 (B) $2\frac{1}{8}$
 (C) $2\frac{3}{8}$
 (D) $2\frac{1}{2}$
 (E) $2\frac{5}{8}$



20. Which polar equation best represents the graph for $0 \leq \theta \leq 2\pi$?

- (A) $r = 8\theta$
 (B) $r = \theta^2$
 (C) $r = \sin 4\theta$
 (D) $r = \cos 4\theta + \frac{1}{4}$
 (E) $r = \sin 8\theta$



21. In logic we use \wedge for “and,” \vee for “or,” and \neg for “not.” Which of the following is logically equivalent to $(P \vee Q) \wedge (Q \vee R)$?

- (A) $P \wedge R$
 (B) $(P \wedge R) \vee Q$
 (C) $(P \wedge Q) \vee (Q \wedge R)$
 (D) $(\neg Q) \wedge (P \vee R)$
 (E) $P \wedge Q$

22. An operation on a row of seven circles, where each circle is either black or white, consists of choosing any **two** of the circles and changing the colors of each of them (i.e., from black to white, or from white to black).



Which of the following rows of circles cannot be obtained by any repeated application of such operations upon the row above?

- (A) ○ ● ○ ○ ● ○ ●
 (B) ● ○ ● ● ● ● ○
 (C) ● ● ● ● ● ● ●
 (D) ○ ○ ○ ○ ● ● ●
 (E) ● ○ ● ○ ● ○ ●

23. The sky is blue because molecules in the atmosphere scatter light. The intensity of scattered light I is inversely proportional to the fourth power of the wavelength λ .

$$I \propto \frac{1}{\lambda^4}$$

Violet light ($\lambda = 400$ nm) is scattered how much more than redder-than-red infrared light ($\lambda = 800$ nm)?

- (A) twice as much
 (B) four times as much
 (C) eight times as much
 (D) sixteen times as much
 (E) thirty-two times as much
24. In the grid each cell contains one of the digits 1 to 5 so that each row and each column has exactly one of each digit. Find the entry in row 3, column 4.

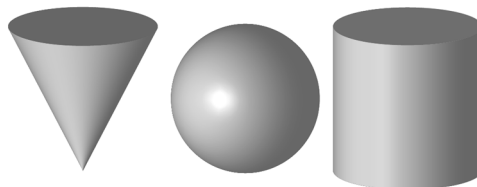
- (A) 1
 (B) 2
 (C) 3
 (D) 4
 (E) 5

1	2			
2				
				5
			5	4

25. For what value of x is there a local minimum of $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 1$?

- (A) $\frac{11}{3}$
 (B) $\frac{8}{5}$
 (C) $-\frac{5}{8}$
 (D) 2
 (E) -3

Questions 26–27: The cone, ball, and cup all have the same height, and for each one the height and diameter are equal.



26. How many cones-full of water will fill the cup?
- (A) 1
 (B) $1\frac{1}{2}$
 (C) 2
 (D) $2\frac{1}{2}$
 (E) 3
27. When the sphere is submerged in the filled cup, water overflows. How many cones-full of water remain in the cup?

- (A) 1
 (B) $1\frac{1}{2}$
 (C) 2
 (D) $2\frac{1}{2}$
 (E) 3

28. Miss Brown said, "I have many brothers and sisters. I am the sixth child and the number of my brothers is at least as large as the number of my sisters." Her younger brother added, "And I have at least twice as many sisters as brothers." How many siblings are there in the Brown family?

- (A) 4
- (B) 6
- (C) 7
- (D) 8
- (E) 9

29. Which is equivalent to $\tan^2 x \cdot \sin^2 x$?

- (A) $\tan^2 x - \sin^2 x$
- (B) $\cot^2 x \cdot \cos^2 x$
- (C) $\cos^2 x$
- (D) $\sec^2 x$
- (E) $\sec^2 x - 1$

30. Manu arrives at the bus stop each day randomly between 7:42 AM and 7:51 AM. The bus arrives at the stop randomly between 7:48 AM and 7:52 AM and immediately departs. How likely is Manu to miss the bus?

- (A) Manu misses the bus $\frac{1}{8}$ of the time.
- (B) Manu misses the bus $\frac{3}{16}$ of the time.
- (C) Manu misses the bus $\frac{1}{3}$ of the time.
- (D) Manu misses the bus $\frac{7}{50}$ of the time.
- (E) Manu misses the bus $\frac{1}{2}$ of the time.

31. In a group of 22 students, 12 like to play basketball, 15 like to play soccer, but two don't like to play either sport. How many like to play both basketball and soccer?

- (A) 7
- (B) 27
- (C) 3
- (D) 5
- (E) 9

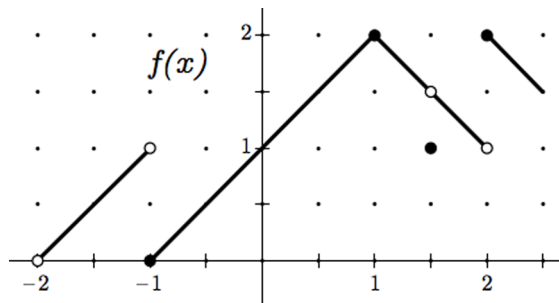
32. Sum all prime numbers between 1 and 100 that are both 1 greater than a multiple of 4 and 1 less than a multiple of 5.

- (A) 118
- (B) 137
- (C) 158
- (D) 187
- (E) 245

33. If $a^x = c^q$ and $c^y = a^z$ (with $a, c > 0$ and $a, c \neq 1$), then which one of the following is true for all q, x, y, z ?

- (A) $xy = qz$
- (B) $\frac{x}{y} = \frac{q}{z}$
- (C) $x + y = q + z$
- (D) $x - y = q - z$
- (E) $x + q = y + z$

34. Select the **one false** statement about $f(x)$.



- (A) $f(-1) = 0$
- (B) $\lim_{x \rightarrow 1} f(x) = 2$
- (C) $\lim_{x \rightarrow 2} f(x)$ does not exist.
- (D) $\lim_{x \rightarrow -1^-} f(x) = 1$
- (E) $\lim_{x \rightarrow 3/2} f(x) = 1$

35. Students in a class are selected at random, one after the other, from a class consisting of 3 boys and 4 girls. What is the probability that girls and boys in the class alternate starting with a girl first?
- (A) $\frac{1}{35}$
 (B) $\frac{34}{35}$
 (C) $\frac{5}{7}$
 (D) $\frac{6}{7}$
 (E) $\frac{32}{35}$
36. Two ladders that are resting on the floor can be made to reach the same vertical height on the wall. One of the ladders is twice as long as the other. If the shorter ladder makes an angle of 60° with the floor, what angle does the longer ladder make with the floor?
- (A) $\sin^{-1} \frac{\sqrt{2}}{2}$
 (B) $\sin^{-1} 2$
 (C) $\sin^{-1} \sqrt{3}$
 (D) $\sin^{-1} \frac{\sqrt{3}}{4}$
 (E) $\sin^{-1} \frac{1}{4}$
37. Cy's company was losing money. As a result Cy received a 25% pay cut. By what percentage must his new salary be raised to bring it back to the original level?
- (A) 25%
 (B) $33\frac{1}{3}\%$
 (C) 40%
 (D) 50%
 (E) 100%
38. What are the last two digits of 11^{22} ?
 Hint: $11^{22} = (10 + 1)^{22}$.
- (A) 01
 (B) 21
 (C) 33
 (D) 51
 (E) 81
39. The number represented as 256 in base 10 has what base 5 representation?
- (A) 128
 (B) 211
 (C) 310
 (D) 512
 (E) 2011
40. If $z = 2 \cos \frac{\pi}{12} + 2i \sin \frac{\pi}{12}$, what is z^4 ?
- (A) 16
 (B) $8 + 8\sqrt{3}i$
 (C) $8\sqrt{3} + 8i$
 (D) $\frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{2}i$
 (E) $(\frac{13}{4} + \sqrt{3}) + (\frac{13}{4} - \sqrt{3})i$